Structured regularization for solving Ecopetrol models with dependent constraints

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Characteristics of models from Ecopetrol

- Many non-convex terms
- Many complementarity constraints
- High nonlinearity from empirical/regressive models
- Hidden dependent equations in the model

Ecopetrol build models but had difficulties to optimize them

We detected that there indeed are dependent constraints within the problems and solved them successfully with structured regularized ipopt

	# var	# con	# dependent constraints	Conventional Ipopt	Structured regularized Ipopt
Problem 1	833	772	5	failed with singular matrix	optimum found #iter = 75
Problem 2	5176	4702	83	failed with ≥ 3000 iter.	optimum found #iter = 399

Basic Barrier NLP

General NLP

$$\begin{aligned} &\min f(x) \\ &s.t.\quad c(x) = 0 \\ &\qquad\quad x \geq 0 \end{aligned}$$

$\Rightarrow$

Barrier NLP

$$\begin{aligned} &\min \varphi_\mu(x) = f(x) - \mu \sum_{i=1}^{n} \ln x^{(i)} \\ &s.t. \qquad\qquad c(x) = 0 \end{aligned}$$

Primal-dual Equations

$$\begin{aligned} \nabla f(x) + A\lambda - \nu &= 0 \\ c(x) &= 0 \\ XVe - \mu e &= 0 \end{aligned}$$

$$A = \nabla c(x) \qquad \nu = \mu X^{-1} e$$

KKT matrix

$$\begin{bmatrix} W_k + \Sigma_k & A_k \\ A_k^T & 0 \end{bmatrix} \begin{bmatrix} d_k^x \\ \lambda_{k+1} \end{bmatrix} = - \begin{bmatrix} \nabla \varphi_\mu(x_k) \\ c(x_k) \end{bmatrix}$$

$$W_k = \nabla_{xx}^2 L_k \qquad \Sigma_k = X_k^{-1} V_k \qquad L(x, \lambda, \mu) = f(x) + c(x)^T \lambda - \nu^T x$$

Structured regularization

Conventional correction

$$\begin{bmatrix} W_k + \Sigma_k & A_k \\ A_k^T & 0 \end{bmatrix} \begin{bmatrix} d_k^x \\ \lambda_{k+1} \end{bmatrix} = - \begin{bmatrix} \nabla \varphi_\mu(x_k) \\ c(x_k) \end{bmatrix} \qquad \begin{bmatrix} W_k + \Sigma_k & A_k \\ A_k^T & -\delta_c I \end{bmatrix} \begin{bmatrix} d_k^x \\ \lambda_{k+1} \end{bmatrix} = - \begin{bmatrix} \nabla \varphi_k \\ c_k \end{bmatrix}$$

Structured regularization

$$A = \left[A_I \mid A_D \right] \qquad LA^T = \begin{bmatrix} U_1 & U_2 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} W + \Sigma & \begin{bmatrix} U_1^T & 0 \\ U_2^T & 0 \end{bmatrix} \\ \begin{bmatrix} U_1 & U_2 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & \boxed{\delta_D I} \end{bmatrix} \end{bmatrix} \begin{bmatrix} d_x \\ \lambda_I \\ \lambda_D \end{bmatrix} = - \begin{bmatrix} \nabla \varphi_\mu \\ (Lc)_I \\ (Lc)_D \end{bmatrix}$$

$$-\delta_D \lambda_D + (Lc)_D = 0$$

$$\delta_D >> (Lc)_D \;\Rightarrow\; \lambda_D \to \varepsilon$$

$$\begin{bmatrix} W + \Sigma & A_I \\ A_I^T & 0 \end{bmatrix} \begin{bmatrix} d^x \\ \lambda_I \end{bmatrix} = - \begin{bmatrix} \nabla \varphi_\mu \\ c_I \end{bmatrix}$$

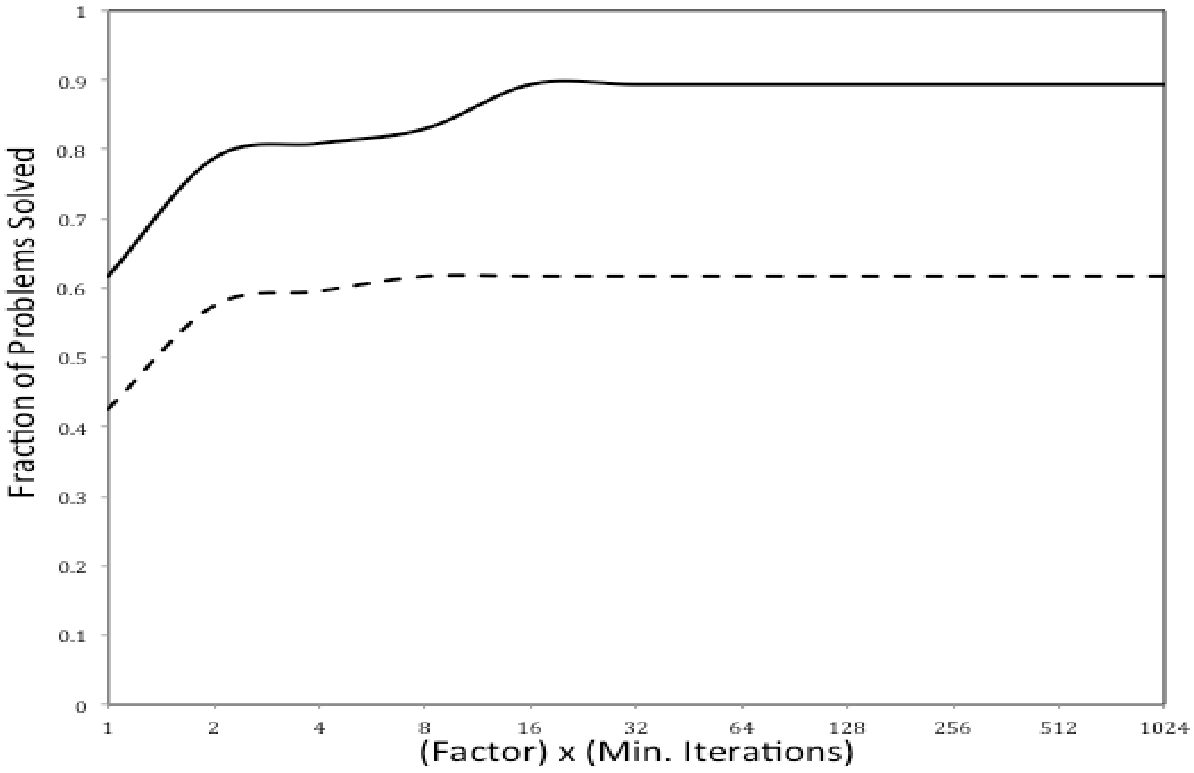
Inertia Correction

To guarantee a descent direction and maintain positive curvature, the Hessian may also need to be regularized

$$\begin{bmatrix} W+\Sigma+\delta_H I & \begin{bmatrix} U_1^T & 0 \\ U_2^T & 0 \end{bmatrix} \\ \begin{bmatrix} U_1 & U_2 \\ 0 & 0 \end{bmatrix} & \boxed{\begin{bmatrix} 0 & 0 \\ 0 & \delta_D I \end{bmatrix}} \end{bmatrix} \begin{bmatrix} d_x \\ \lambda_I \\ \lambda_D \end{bmatrix} = -\begin{bmatrix} \nabla\varphi_\mu \\ (Lc)_I \\ (Lc)_D \end{bmatrix}$$

$$\begin{bmatrix} W+\Sigma+\delta_H I & A \\ A^T & \boxed{-M} \end{bmatrix} \begin{bmatrix} d^x \\ \lambda \end{bmatrix} = -\begin{bmatrix} \nabla\varphi_\mu \\ c \end{bmatrix}$$

Performance of the method



Structured regularization Ipopt

Conventional Ipopt

Conclusions

- In order to solve problems with dependent constraints, a structured regularization method has been developed for barrier NLP solvers (e.g. IPOPT).
- The method improves the performances of IPOPT significantly to overcome the difficulties for handling dependency of the models
- The problems formulated by Ecopetrol have been solved successfully and efficiently.